Waveguide-QED-Based Photonic Quantum Computation

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We propose a new scheme for scalable quantum computation using flying qubits—propagating photons in a one-dimensional waveguide—interacting with matter qubits. Photon-photon interactions are mediated by the coupling to a three- or four-level system, based on which photon-photon π-phase gates (Controlled-NOT) can be implemented for universal quantum computation. We show that high gate fidelity is possible given recent dramatic experimental progress in superconducting circuits and photonic-crystal waveguides. The proposed system can be an important building block for future on-chip quantum networks.

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Quantum computers hold great promise for outperforming any classical computer in solving certain problems such as integer factorization [1], as well as in efficiently simulating quantum many-body systems [2, 3]. While quantum computation schemes often encode information in stationary qubits such as atoms, trapped ions, quantum dots and superconducting qubits [4], flying qubits—photons—have several appealing features as carriers of quantum information [4, 5]. Most importantly, photons have long coherence times because they rarely interact, and yet can be readily manipulated at the single photon level using linear optics. Furthermore, photonic quantum computation is potentially scalable in view of the recent controlled generation of single-photon pulses [6–10] and demonstration of stable quantum memories [11, 12]. However, weak photon-photon interactions makes it very challenging to realize the two-qubit gates necessary for universal computation between single-photons. Several schemes have been proposed to circumvent this difficulty. The linear optics scheme [13] uses quantum interference between qubit photons and auxiliary photons to generate an effective nonlinear interaction between qubit photons. Other approaches include employing trapped atoms in a cavity [14, 15] or Rydberg atoms [16, 17] to realize two-qubit gates.

In this work, we propose an alternative scheme for photonic quantum computation: using strong coupling between local emitters and photons in a one-dimensional (1D) waveguide. Because of recent tremendous experimental progress [8–10, 18–24], 1D waveguide systems are becoming promising candidates for quantum information processing. A variety of capabilities have been proposed [25–32], particularly at the single photon level, yet protocols compatible with current waveguide setups for some important tasks, notably two-qubit gates, have yet to emerge. In our proposal, we construct photonic two-qubit gates solely based on scattering in a waveguide system that is accessible in current experiments. Compared with the cavity approach, our setup is simplified and avoids the complexity of stabilizing the resonance between the cavity modes and the atom. The gate has a wide bandwidth, and its operation time is determined solely by the coupling strength. Combining the simplicity of the system and the scalability of photons, our waveguide-QED-based scheme opens a new avenue towards scalable quantum computation and distributed quantum networks [33] in a cavity-free setting.

The photonic qubits are encoded in the frequency degree of freedom, $|\omega_0\rangle$ and $|\omega_1\rangle$, for simplicity; a straightforward generalization of our scheme is applicable to polarization qubits. Single photons can be generated from the emission of quantum dots [8, 9, 23] or using circuit-QED systems [10], and single-qubit rotations can be realized using a Mach-Zehnder interferometer [34, 35]. Hence, we focus on two-qubit gates and, in particular, a π-phase (Controlled-NOT) gate. We consider a semi-infinite 1D waveguide side-coupled to a four-level system that is located a distance a from the end (Fig. 1). Such a setup can be realized in a variety of experimental systems using superconducting transmission lines [21, 22], photonic-crystal waveguides [9], hollow fibers with trapped cold atoms [19], or plasmonic nanowires [18]. We now show that a π-phase gate between two photons A and B can be realized by reflecting them from the end of the waveguide.

The Hamiltonian of the system (Fig. 1) is given by

$$H = H_{\rm wg} + \sum_{i=2}^{4} \hbar(\Omega_i - i\Gamma_i'/2)\sigma_{ii}$$
$$+ \sum_{\alpha=R,L} \int dx\, \hbar V\, \delta(x)[a_\alpha^\dagger(x)(\sigma_{12}+\sigma_{32}+\sigma_{34}) + \text{h.c.}],$$
$$H_{\rm wg} = \int dx \frac{\hbar c}{i}\left[a_R^\dagger(x)\frac{d}{dx}a_R(x) - a_L^\dagger(x)\frac{d}{dx}a_L(x)\right], \quad (1)$$

where $a_{R,L}$ are the waveguide modes, $\sigma_{ij} \equiv |i\rangle\langle j|$, and we use the energy level of ground state $|1\rangle$ as the energy reference. An imaginary term models the loss of the excited state at rate Γ_i'. The decay rate to the waveguide continuum is $\Gamma = 2V^2/c$, where c is the group velocity of photons. For our gate operation, we require that the transitions $1 \rightarrow 2$ and $3 \rightarrow 4$ have the same frequency $\Omega_{12} = \Omega_{34}$ (where $\Omega_{ij} \equiv \Omega_j - \Omega_i$); in contrast, the frequency of the

$3\rightarrow2$ transition should be distinctly different, satisfying $|\Omega_{32}-\Omega_{12}|\gg\Gamma$. In addition, for simplicity, we assume that (i) transitions $1\rightarrow2$, $3\rightarrow2$, and $3\rightarrow4$ have the same coupling strength Γ to the waveguide modes, (ii) state 3 is metastable with loss rate $\Gamma_3'=0$, and (iii) states 2 and 4 have the same loss rate $\Gamma'\equiv\Gamma_2'=\Gamma_4'$. None of these additional assumptions are essential. Here, we set $\hbar=c=1$.

The photon qubit consists of two distinct frequencies. Frequency ω_1 is chosen to be on resonance with the transitions $1\rightarrow2$ and $3\rightarrow4$, *i.e.* $\omega_1=\Omega_{12}$. In contrast, ω_0 is far off resonance from all of the atomic transitions—an ω_0 photon does not interact with the four-level system (4LS). The 4LS is initialized in $|1\rangle$.

As illustrated in Fig. 1, a π-phase gate between photon pulses A and B can be realized via the following three steps. (1) Trapping: the first qubit photon A is sent into the system. If A is in state $|\omega_1\rangle$, it is trapped, an auxiliary photon C at frequency Ω_{32} is emitted, and the 4LS is put into $|3\rangle$. (2) π-phase: a second qubit photon B is sent into the system; it gains a π-phase if it is in the $|\omega_1\rangle$ state and the 4LS is in state $|3\rangle$. (3) Retrieval: by time reversal arguments, sending in an auxiliary photon C retrieves the trapped photon if the 4LS is in state $|3\rangle$. Therefore, only when both photons A and B are in state $|\omega_1\rangle$ is a π-phase generated by their interaction with the 4LS. Note that this scheme is passive in the sense that no external manipulation of the 4LS is required. We now analyze each step.

Step 1—Trapping. For an incoming single photon A in mode $|\omega_A\rangle$ and initial state $|1\rangle$ of the 4LS, the output state of the system, obtained from wavefunction matching [32], is

$$|\phi_1^{\rm out}(\omega_A)\rangle=r_{11}(\omega_A)|\omega_A\rangle\otimes|1\rangle+r_{13}(\omega_A)|\tilde{\omega}_A\rangle\otimes|3\rangle, \quad (2)$$

where

$$\begin{aligned}\tilde{\omega}&=\omega-\Omega_{13},\\ r_{11}(\omega)&=e^{2i\omega a}\frac{-\Omega_{12}+\frac{i\Gamma'}{2}+\omega-\frac{i\Gamma}{2}[e^{2i\tilde{\omega}a}-e^{-2i\omega a}]}{\Omega_{12}-\frac{i\Gamma'}{2}-\omega+\frac{i\Gamma}{2}[e^{2i\tilde{\omega}a}+e^{2i\omega a}-2]},\\ r_{13}(\omega)&=\frac{(i\Gamma/2)(e^{2i\omega a}-1)[e^{2i\tilde{\omega}a}-1]}{\Omega_{12}-\frac{i\Gamma'}{2}-\omega+\frac{i\Gamma}{2}[e^{2i\tilde{\omega}a}+e^{2i\omega a}-2]}\,. \quad (3)\end{aligned}$$

We first illustrate the operation principle for the lossless case $\Gamma'=0$ and then later analyze the effect of loss in detail. We assume that the the key condition $2(\Omega_{12}+\Omega_{32})a=2n_1\pi$ is satisfied; in addition, we can make the trivial choice $2\omega_0 a=(2n_0+1)\pi$ (n_0,n_1 are integers). Then, if the incoming qubit photon-A is in mode $|\omega_0\rangle$, $r_{11}(\omega_0)=1$ and $r_{13}(\omega_0)=0$ because ω_0 is far off resonance from all the transitions. Hence, it will reflect from the system without change, leaving the 4LS in $|1\rangle$. On the other hand, if photon-A is in mode $|\omega_1\rangle$, the on-resonance interaction with the $1\rightarrow2$ transition gives $r_{11}(\omega_1)=0$ and $r_{13}(\omega_1)=-1$. As a result, it will

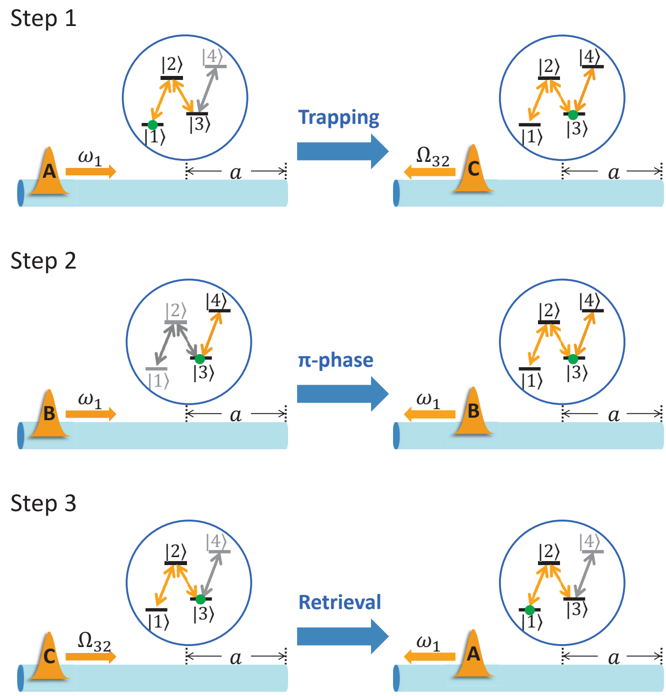

FIG. 1. Steps of the gate operation: 1) trapping; 2) π-phase; 3) retrieval. The gate sequences here illustrate the case of both photons A and B initially being in state $|\omega_1\rangle$. The left and right sides show the initial and final states, respectively. Inactive transitions in each step are gray-colored.

be trapped and stored in level $|3\rangle$ of the 4LS, emitting an auxiliary C-photon at frequency $\omega_1-\Omega_{13}=\Omega_{32}$.

Step 2—π-phase. Now send in the second qubit, photon-B in mode $|\omega_B\rangle$. The output state after scattering reads

$$\begin{aligned}|\phi_2^{\rm out}(\omega_A,\omega_B)\rangle&=r_{13}(\omega_A)R_3(\omega_B)|\tilde{\omega}_A\rangle|\omega_B\rangle\otimes|3\rangle\\&\quad+r_{11}(\omega_A)|\omega_A\rangle\otimes|\phi_1^{\rm out}(\omega_B)\rangle, \quad (4)\end{aligned}$$

where

$$R_3(\omega)=\frac{-(\Omega_{12}-\frac{i\Gamma'}{2}-\omega)e^{2i\omega a}+\frac{i\Gamma}{2}(1-e^{2i\omega a})}{\Omega_{12}-\frac{i\Gamma'}{2}-\omega-\frac{i\Gamma}{2}(1-e^{2i\omega a})}. \quad (5)$$

Here, we neglect the transition $3\rightarrow2$ because $\omega_{0,1}$ is chosen to be far detuned from Ω_{32}. If photon-B is in mode $|\omega_0\rangle$, it is far off resonance from the transitions, and, using the same value of a as above, $r_{11}(\omega_0)=R_3(\omega_0)=1$ while $r_{13}(\omega_0)=0$. Hence, the output state in this case is $|\phi_1^{\rm out}(\omega_A)\rangle\otimes|\omega_B\rangle$—photon-$B$ is unaffected. However, if photon-B is in $|\omega_1\rangle$, the state after scattering is

$$\begin{aligned}|\phi_2^{\rm out}(\omega_A,\omega_B=\omega_1)\rangle&=|\phi_{AB}(\omega_A,\omega_B)\rangle\otimes|3\rangle,\\ |\phi_{AB}(\omega_A,\omega_B)\rangle&=r_{13}(\omega_A)R_3(\omega_B)|\tilde{\omega}_A\rangle|\omega_B\rangle\\&\quad+r_{11}(\omega_A)r_{13}(\omega_B)|\omega_A\rangle|\tilde{\omega}_B\rangle. \quad (6)\end{aligned}$$

Two possible outcomes exist: (i) if the 4LS is in state $|1\rangle$ after step 1, photon-B will be trapped, but (ii) if the 4LS

is in state $|3\rangle$, photon-B is on resonance with transition $3 \to 4$ and gains a π-phase shift $[R_3(\omega_1) = e^{i\pi}]$. The 4LS being in state $|3\rangle$ is, of course, conditioned upon photon-A in Step 1 being in $|\omega_1\rangle$.

Step 3—Retrieval. Finally, by sending in the auxiliary photon-C in mode $|\omega_C = \Omega_{32}\rangle$, we retrieve the trapped photon—either photon-A from Step 1 or photon-B from Step 2. This is the time-reversal process of photon trapping. The final state after all three steps reads

$$|\phi_3^{\rm out}(\omega_A, \omega_B, \omega_C)\rangle = r_{11}(\omega_A) r_{11}(\omega_B) |\omega_A\rangle|\omega_B\rangle|\phi_1^{\rm out}(\omega_C)\rangle + |\phi_{AB}(\omega_A, \omega_B)\rangle \otimes |\varphi_3^{\rm out}(\omega_C)\rangle, \quad (7)$$

where $|\phi_{AB}(\omega_A, \omega_B)\rangle$ is defined in Eq. (6) and

$$|\varphi_3^{\rm out}(\omega_C)\rangle = \Big[r_{31}(\omega_C)|\omega_C + \Omega_{13}\rangle + r_{33}(\omega_C)|\omega_C\rangle\Big] \otimes |1\rangle,$$

$$r_{33}(\omega) = e^{2i\omega a} \frac{-\Omega_{32} + \frac{i\Gamma'}{2} + \omega - \frac{i\Gamma}{2}[e^{2i(\omega+\Omega_{13})a} - e^{-2i\omega a}]}{\Omega_{32} - \frac{i\Gamma'}{2} - \omega + \frac{i\Gamma}{2}[e^{2i(\omega+\Omega_{13})a} + e^{2i\omega a} - 2]},$$

$$r_{31}(\omega) = \frac{(i\Gamma/2)(e^{2i\omega a} - 1)[e^{2i(\omega+\Omega_{13})a} - 1]}{\Omega_{32} - \frac{i\Gamma'}{2} - \omega + \frac{i\Gamma}{2}[e^{2i(\omega+\Omega_{13})a} + e^{2i\omega a} - 2]}\ . \quad (8)$$

By tracing out the auxiliary photon-C at frequency Ω_{23}, we obtain the final state $|\phi_f(\omega_A = \omega_i, \omega_B = \omega_j) = (-1)^{ij}|\omega_i\rangle|\omega_j\rangle \otimes |1\rangle$, $i, j = 0$ or 1.

Thus we see that the above three steps combined give rise to the desired π-phase gate

$$U_{AB} = \exp\left\{i\pi|\omega_1\rangle_A\langle\omega_1| \otimes |\omega_1\rangle_B\langle\omega_1|\right\}. \quad (9)$$

A valuable additional benefit of this scheme is that the combination of trapping and retrieval alone (Steps 1 and 3) realizes a quantum memory for single-photon storage, providing an important element for use in a quantum network.

We now analyze the gate performance by considering photon pulses with a finite spectral width σ and including atomic loss ($\Gamma' > 0$). In particular, we consider Gaussian input pulses A, B, and C centered at frequencies ω_1, ω_1, and Ω_{32}, respectively:

$$|\phi_{A,B}\rangle = \int d\omega_{A,B}\, g_\sigma(\omega_{A,B} - \omega_1)|\omega_{A,B}\rangle,$$

$$|\phi_C\rangle = \int d\omega_C\, g_\sigma(\omega_C - \Omega_{32})|\omega_C\rangle, \quad g_\sigma(\omega) \propto e^{-\frac{\omega^2}{2\sigma^2}}. \quad (10)$$

The corresponding temporal width is $\Delta T = 1/(2\sigma)$. After the scattering, the final state of the system is $|\phi_f\rangle = \int d\omega_A d\omega_B d\omega_C\, g_\sigma(\omega_A) g_\sigma(\omega_B) g_\sigma(\omega_C) |\phi_3^{\rm out}(\omega_A, \omega_B, \omega_C)\rangle$. The fidelity of the photon-atom gate is given by

$$F \equiv |\langle\psi|\phi_f\rangle|^2, \quad (11)$$

where $|\psi\rangle = -|\phi_A\rangle|\phi_B\rangle|\phi_C\rangle \otimes |1\rangle$ is the target state.

The atomic loss is characterized by introducing the effective Purcell factor $P = \Gamma/\Gamma'$. We note that large values of P (> 20) have been demonstrated in recent experiments using either superconducting circuits [24], photonic-crystal waveguides [9], or semiconductor nanowires [36]. To quantify the effect of loss, we define the probability of leakage, P_ℓ, to be the probability of losing the photon during the operation through spontaneous emission

$$P_\ell \equiv 1 - |\langle\phi_f|\phi_f\rangle|^2. \quad (12)$$

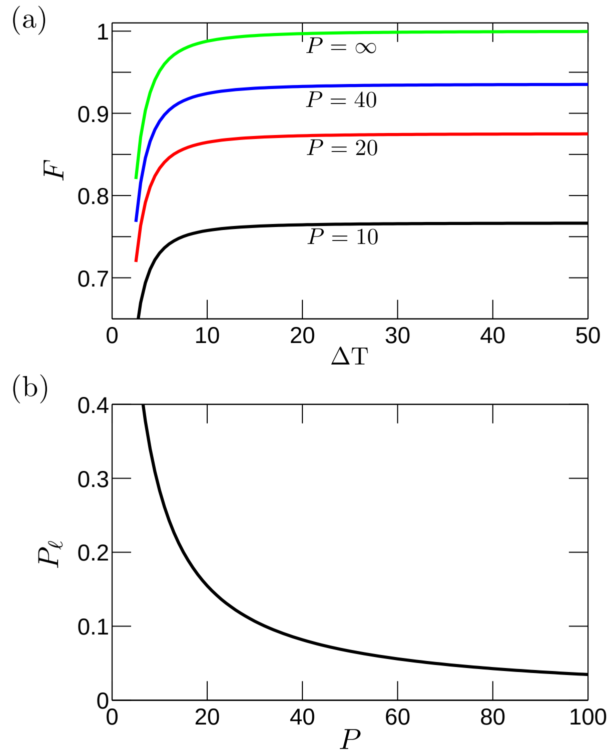

FIG. 2. Fidelity and leakage error of the photon-photon π-phase gate. (a) Fidelity F as a function of the pulse width ΔT (in units of Γ^{-1}) for $P = 10, 20, 40, \infty$. (b) The leakage probability P_ℓ as a function P with $\Delta T = 10\Gamma^{-1}$.

Figure 2(a) shows the fidelity of our scheme as a function of the pulse width ΔT. For a short pulse, the spectral width is large, and so the fidelity is limited by the large frequency variation. As ΔT increases to $10\Gamma^{-1}$, the fidelity starts to saturate and is only limited by the atomic loss. A fidelity of 86% and 94% can be achieved for $P = 20$ and $P = 40$, respectively. Figure 2(b) shows that the leakage error decays rapidly as P increases and can be as small as a few percent for P approaching 100, which is feasible in the near future given the rapid experimental advances in 1D waveguide systems. Such a leakage error is acceptable, especially since it can be efficiently corrected by concatenated coding [1, 14].

We now make a rough estimate of the gate operation time. Since the gate fidelity is insensitive to the pulse width variation once ΔT is sufficiently large [Fig. 2(a)], we choose $\Delta T = 10\Gamma^{-1}$ for practical estimation. Using a superconducting circuit as an example, we estimate the duration of our photon-photon π-phase gate

to be $30\Gamma^{-1} \sim 300\,\text{ns}$ for a superconducting qubit with $\Gamma = 2\pi \times 100\,\text{MHz}$ [24]. Such an operation time is compatible with current qubit coherence times, which are on the order of $1\,\mu\text{s}$ [37]

An important property of the 4LS two photon gate introduced here is that no manipulation of the 4LS is needed—it is a passive scheme. However, if one wishes to consider scenarios involving active manipulation of the local quantum system, a π-phase gate using only a three level system (3LS) is possible by adapting the cavity-based proposal of Ref. 14. First, one constructs a π-phase gate between a photon qubit and the local qubit (3LS). Then, using the photon-atom π-phase gate as a building block, a π-phase gate between two photons A and B can be implemented by sending them into the system successively. This proposal has the additional advantage of naturally realizing a photon-atom π-phase gate, which can be used to entangle distant quantum nodes in a large quantum network [38]. On the other hand, the 3LS scenario does not realize a quantum memory, an important advantage of the 4LS scenario. Details of the 3LS scheme can be found in the Supplementary Material [39]

In summary, we demonstrate that two-qubit gates for scalable photonic quantum computation can be designed in 1D waveguide-QED systems. Our waveguide-based proposal has several potential advantages over quantum computation based on cavity photons or stationary qubits. The operation time here is limited only by the coupling strength, while in the cavity case the cavity line width is the bottleneck. Also, our scheme does not require fine tuning of the interaction time, which is often a significant source of error. Overall, the system proposed here can be an important building block for future on-chip quantum networks: taking superconducting circuits as an example, we can envision such a network with (i) single photons generated using microwave resonators [10], (ii) photons stored in quantum memories formed from the 4LS proposed above, (iii) photon flow regulated by single-photon routers [24], and (iv) two-photon operations realized by our 4LS-waveguide system.

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Supplementary Material for "Waveguide-QED-Based Photonic Quantum Computation"

In this Supplementary Material, we present an alternative implementation of a photon π-phase gate based on a three-level system (3LS). In this realization (Fig. S1), we consider a semi-infinite 1D waveguide side-coupled to a 3LS, which is located a distance a from the end. Such a system could be realized in a variety of experimental systems [1–5]. A π-phase gate between two photons A and B is realized by reflecting them from the end of the waveguide.

The Hamiltonian of the system is given by

$$H_1 = H_{\rm wg} + H_{ee} + \sum_{\alpha=R,L} \int dx \hbar V \delta(x)[a_\alpha^\dagger(x)\sigma_{ge} + \text{h.c.}],$$
$$H_{\rm wg} = \int dx \frac{\hbar c}{i}\Big[a_R^\dagger(x)\frac{d}{dx}a_R(x) - a_L^\dagger(x)\frac{d}{dx}a_L(x)\Big], \tag{S1}$$

where $H_{ee} = \hbar(\Omega_{eg} - i\Gamma'/2)\sigma_{ee}$ and $\sigma_{ij} \equiv |i\rangle\langle j|$. The transition from the ground state $|g\rangle$ to the excited state $|e\rangle$ couples to the waveguide modes ($a_{R,L}$); an imaginary term models the loss of the excited state at rate Γ'. The decay rate to the waveguide continuum is $\Gamma = 2V^2/c$, where c is the group velocity of photons. For simplicity, we set $\hbar = c = 1$. Note that the metastable state $|s\rangle$ does not appear in the Hamiltonian as it is decoupled from the waveguide; however, its presence is essential since $|g\rangle$ and $|s\rangle$ form the atomic qubit. As in the main text, we consider a photonic qubit coded in the frequency domain.

Our first step is to realize a photon-atom π-phase gate. Consider a single incoming photon with frequency ω. In the case that the three-level system is in $|g\rangle$, we obtain the output state by imposing a hard-wall boundary condition at the end of the waveguide, thus giving

$$|\phi_g^{\rm out}(\omega)\rangle = r_g(\omega)|\omega\rangle_L,$$
$$r_g(\omega) = \frac{-(\Delta - \frac{i\Gamma'}{2})e^{2i\omega a} + \frac{i\Gamma}{2}(1-e^{2i\omega a})}{\Delta - \frac{i\Gamma'}{2} - \frac{i\Gamma}{2}(1-e^{2i\omega a})}, \tag{S2}$$

where $\Delta = \Omega_{eg} - \omega$ is the detuning. We first illustrate the operation principle for the lossless case $\Gamma' = 0$ and then later analyze the effect of loss in detail. In the lossless case, we always have perfect reflection—$|r_g(\omega)|^2 = 1$—because

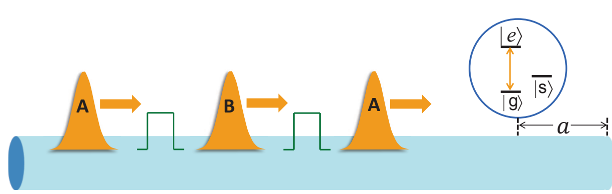

FIG. S1. Schematic diagram of the operation sequence of the π-phase gate between two photons A and B. The three-level system (3LS) is located a distance a from the end of the semi-infinite waveguide and is initialized in an equal superposition of the g and s states. A and B are reflected successively from the semi-infinite waveguide coupled to the 3LS. Between the reflections, single-qubit rotation pulses (green rectangles) are applied to the atomic qubit made of states $|g\rangle$ and $|s\rangle$.

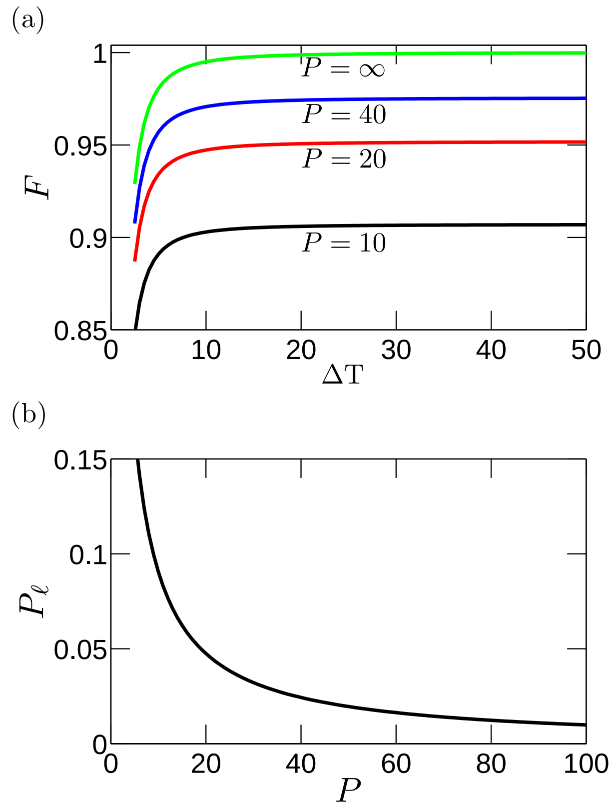

FIG. S2. Fidelity and leakage error of the photon-atom gate in scheme 1. (a) Fidelity F as a function of the pulse width ΔT for four different cases. (b) The leakage probability P_ℓ as a function of the effective Purcell factor P with a pulse width $\Delta T = 10\Gamma^{-1}$.

the waveguide is semi-infinite. Choosing two frequencies $\omega_1 = \Omega_{eg}$ and $|\omega_0 - \Omega_{eg}| \gg \Gamma$, we have $r_g(\omega_1) = -1$ and $r_g(\omega_0) = -e^{2i\omega_0 a}$. On the other hand, if the qubit is in $|s\rangle$, the photon gains a trivial phase shift $r_s(\omega) = -e^{2i\omega a}$. Therefore, under the conditions $2\omega_0 a = (2n_0+1)\pi$ and $2\Omega_{eg}a = (2n_1+1)\pi$ with $n_0, n_1 \in \mathbb{Z}$, we realize a π-phase gate between the photonic qubit ($|\omega_0\rangle$, $|\omega_1\rangle$) and the atomic qubit ($|g\rangle$, $|s\rangle$):

$$\begin{aligned} r_g(\omega_0) = r_s(\omega_0) = r_s(\omega_1) = -r_g(\omega_1) = 1, \\ U_{\text{photon-atom}} = \exp\left\{i\pi|\omega_1\rangle\langle\omega_1| \otimes |g\rangle\langle g|\right\}. \end{aligned} \tag{S3}$$

Using the photon-atom π-phase gate as a building block, we can implement a π-phase gate between two photons A and B as in the cavity-based proposal of Ref. 6. First, initialize the atom in the state $|\phi_a\rangle = (|g\rangle + |s\rangle)/\sqrt{2}$. Next, send in photon-$A$ followed by a $\pi/2$ rotation on the atom. Third, send in photon-B followed by a $-\pi/2$ rotation on the atom. Finally, send in photon-A again. This procedure produces a π-phase gate

$$U_{AB} = \exp\left\{i\pi|\omega_1\rangle_A\langle\omega_1| \otimes |\omega_1\rangle_B\langle\omega_1|\right\} . \tag{S4}$$

Our scheme closely resembles the cavity-based proposal [6], but we rely on a different mechanism to generate the π-phase shift in a cavity-free setting. As for the 4LS in the main text, this phase gate requires fine tuning so that, as noted above, $2\Omega_{eg}a = (2n_1+1)\pi$; this is possible using superconducting qubits, for instance, for which the transition frequencies can be easily tuned using external magnetic flux [7].

To analyze the gate performance in this scheme, we consider photon pulses with a finite spectral width σ and include atomic loss ($\Gamma' > 0$). In particular, we consider a Gaussian input pulse centered at frequency ω_1:

$$|\phi_i\rangle = \int d\omega g_\sigma(\omega)|\omega\rangle \otimes |\phi_a\rangle , \tag{S5}$$

$$g_\sigma(\omega) \propto \exp\{-(\omega-\omega_1)^2/2\sigma^2\}. \tag{S6}$$

The temporal width is $\Delta T = 1/(2\sigma)$. After the scattering, the final state of the system is

$$|\phi_f\rangle = \int d\omega g_\sigma(\omega) \times \big\{ r_g(\omega)|\omega\rangle \otimes |g\rangle + r_s(\omega)|\omega\rangle \otimes |s\rangle \big\}/\sqrt{2}. \tag{S7}$$

The fidelity of the photon-atom gate is given by

$$F \equiv |\langle\psi|\phi_f\rangle|^2 = \left| \frac{1}{2} \int d\omega g_\sigma^2(\omega)[r_g(\omega) - r_s(\omega)] \right|^2, \tag{S8}$$

where $|\psi\rangle = |\omega_1\rangle \otimes (-|g\rangle + |s\rangle)/\sqrt{2}$ is the target state. The atomic loss is characterized by introducing the effective Purcell factor $P = \Gamma/\Gamma'$. To measure the effect of loss quantitatively, we define the probability of leakage, P_ℓ, as the probability of losing the photon during the operation through spontaneous emission:

$$P_\ell \equiv 1 - |\langle\phi_f|\phi_f\rangle|^2 = 1 - \left| \int d\omega g_\sigma^2(\omega) \frac{[|r_g(\omega)|^2 + 1]}{2} \right|^2. \tag{S9}$$

Figure S2(a) shows the fidelity F as a function of pulse temporal width ΔT. For a short pulse, the spectral width is large, and so the fidelity is limited by the large frequency variation of the conditional phase $r_g(\omega)$. As ΔT increases to $10\Gamma^{-1}$, the fidelity starts to saturate and is only limited by the atomic loss. A high fidelity ($\geq 95\%$) can be achieved for a practical value of $P \geq 20$. Figure S2(b) shows that the leakage probability decreases quickly as one increases P and is on the order of a few percent for $P \geq 20$. Further improvement in both fidelity and loss can be expected from the rapidly development of 1D waveguide technology and schemes using off-resonance mechanisms to reduce the loss.

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